

# Understanding Measurement Data

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June 04, 2015

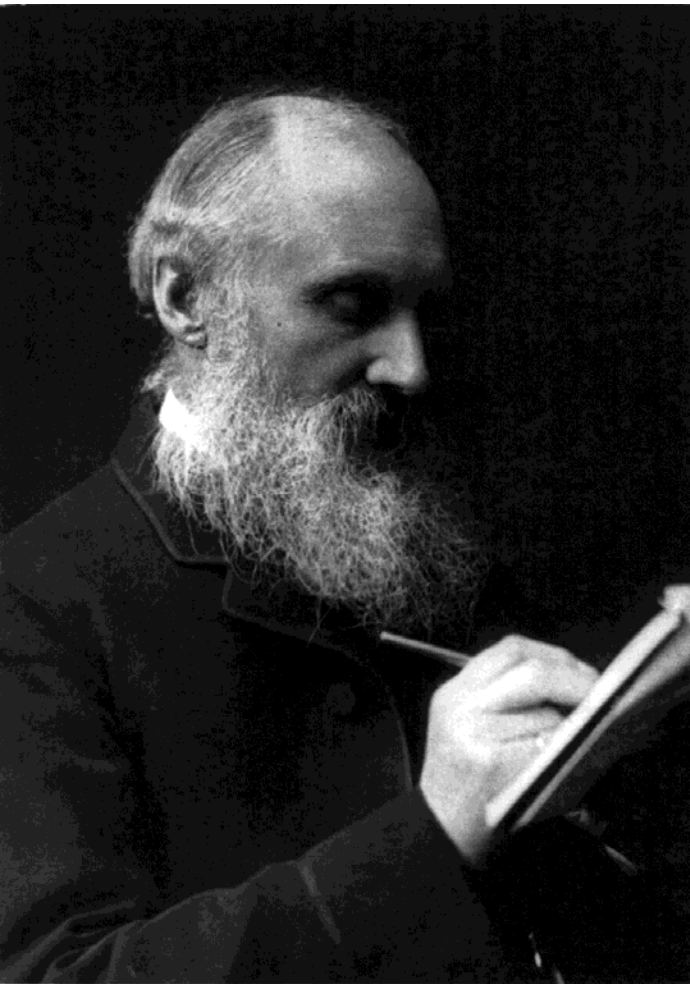
# Understanding Measurement Data

## Topics

- What is Measurement
- “True” Value, Error, Accuracy, Precision, and Uncertainty
- Sources of Data Error in Measurement
- Statistical Treatment of Measurement Errors
- Example of a Calibration Certificate
- Questions

# SIR WILLIAM THOMSON - LORD KELVIN

(1824 – 1907)



...when you can measure what you are speaking about, and express it in numbers, you know something about it.....

[ Quote from Vol. 1, "Electrical Units of Measurement", 1883-05-03 ]

# What is “Measurement”?

- Measurement is the quantitative description of an object or an event.
- Measurements are typically made with some kind of instruments.
- Measurement forms the basis for all disciplines of engineering and sciences.



# Components of Measurement Data

A complete measurement statement is typically presented in three parts:

- Number (magnitude)
- Unit (dimension)
- Uncertainty (quality)

Example: 82.5°C ± 0.1°C at a confidence level of 95%



# Measurement Standards

- The quantitative description of a measurement is obtained by comparison to an established “Standard”.
- Modern measurement quantities are defined in relationship to internally recognized reference standards.



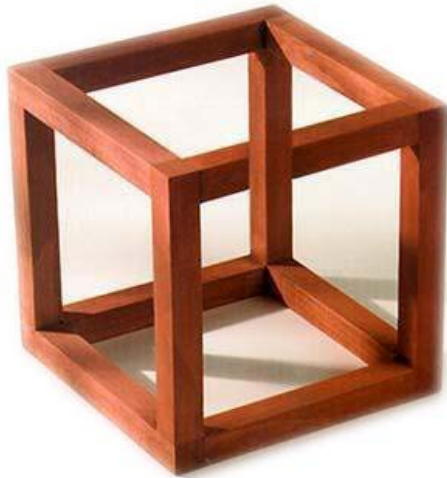
Example:

Canada's national standard for the kilogram is a prized ingot of platinum iridium called K74 housed in a vault at the NRC in Ottawa

# “True” Value, Error, Accuracy, Precision, and Uncertainty

## “True” Value

- “True” value is the measurement result with all sources of error removed.
- It is an idealized concept and that never be realized in practice.



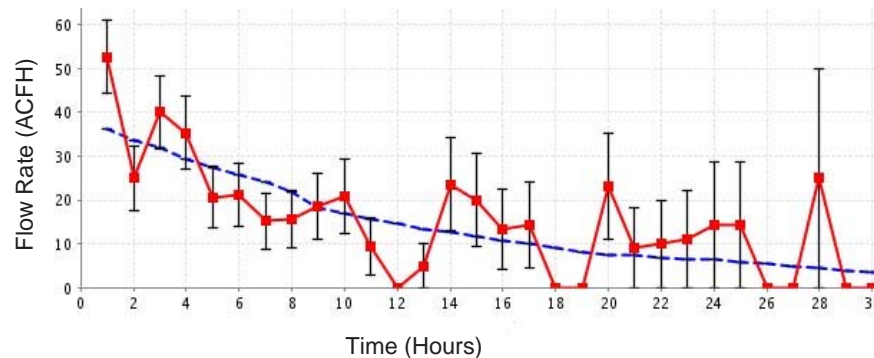
- All measurement are not perfect!!



# Measurement Error

- Measurement Error is the difference between the measured value and the “true” value.
- Measurement Error is typically expressed in percentage of the measured value :

$$Error = \left( \frac{Value_{(Measured)}}{Value_{(True)}} - 1 \right) \times 100\%$$



## Conventional (Orthodox) Definition of Errors

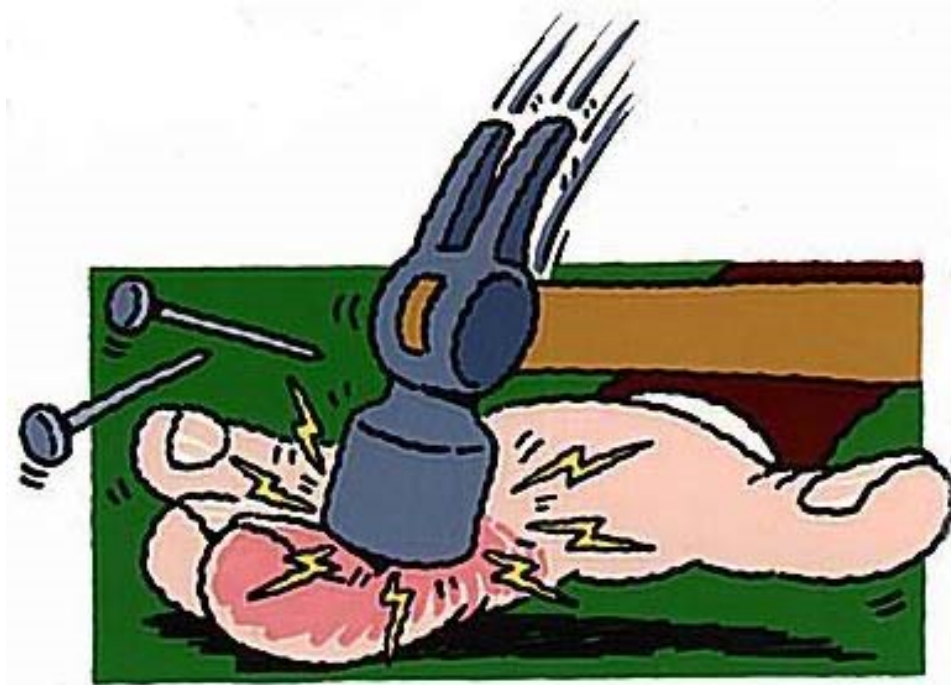
### **Random errors**

Errors that lead to measured values being inconsistent when repeated measures of a constant attribute or quantity are taken.

### **Systematic errors**

Errors that are predictable, and typically constant or proportional to the true value. Systematic errors are sometimes called bias errors.

## Random vs Systematic Errors



# A Newer Approach to Measurement Uncertainty The ISO GUM Terminology

- Promoted by the “ISO Guide to the Expression of Uncertainty in Measurements (1993)”, otherwise known as GUM.
- Adopted by OIML, NIST, NRC, and..... Measurement Canada.
- ISO GUM uses the terms *Type A* and *Type B* Uncertainties instead of the conventional Systematic and Random Errors.

## *Type A and Type B Uncertainty Evaluation*

- *Type A* evaluation of standard uncertainty may be based on any valid statistical method for treating data.
- *Type B* evaluation of standard uncertainty is usually based on scientific judgment using all of the relevant information available.

## *Type A and Type B Uncertainty Evaluation*

The purpose of the *Type A* and *Type B* classification is to indicate the two different ways of evaluating uncertainty components.....it is not meant to indicate that there is any difference in the nature of the components resulting from the two types of evaluation.....

# Accuracy and Precision

- Accuracy refers to how close a measurement is to the “true” value. An accurate measurement has small error.
- Precision is the spread of different readings of a measurement.

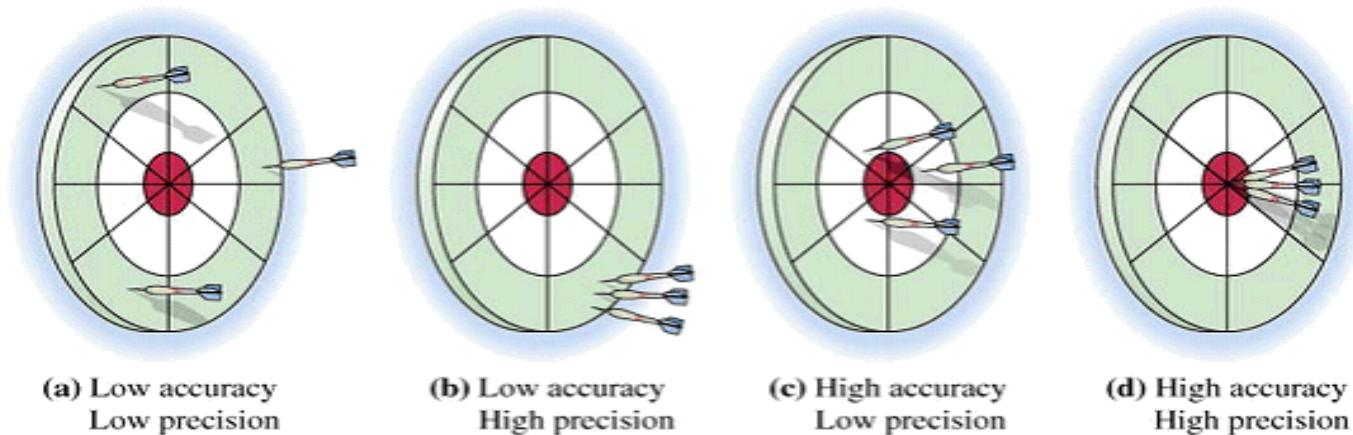
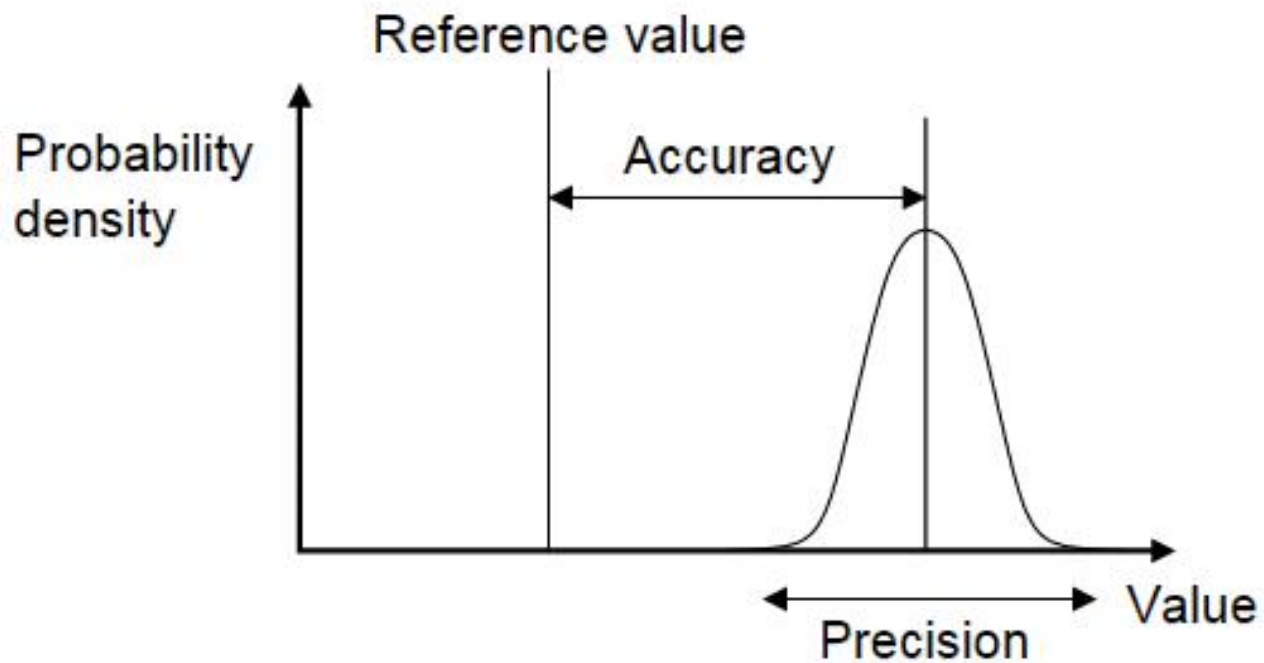


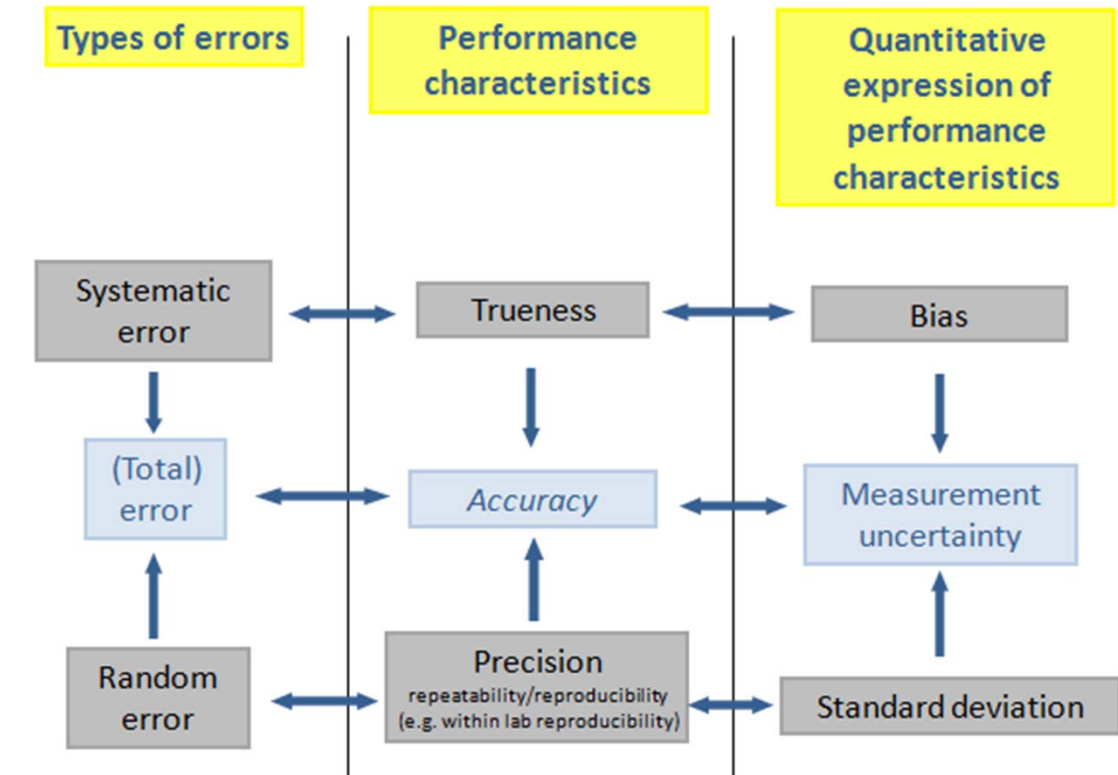
Diagram courtesy of National Instruments publication “Analog Sample Quality: Accuracy, Sensitivity, Precision, and Noise” Apr. 22, 2015

# Accuracy and Precision





# Error, Accuracy, and Uncertainty



Quote from *Accred. Qual. Assur.* by A. Menditto et al 2006, 12, 45

# What is Measurement Uncertainty?

Measurement uncertainty = Quality of a measurement result

Measurement uncertainty must always be expressed with an indication of the level of confidence

## Example

The uncertainty of this measurement is  **$\pm 0.1$  inch** with a **confidence level of 95%** (coverage factor  $k=2$ )

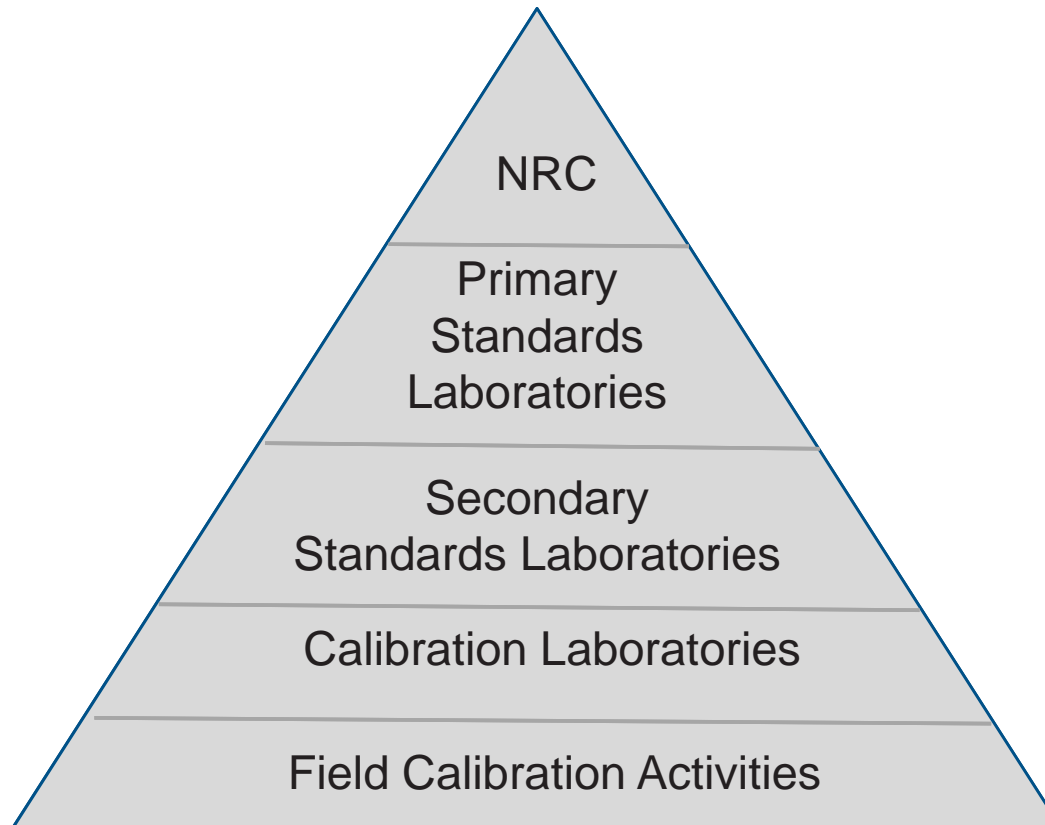
# Traceability and Measurement Uncertainty

Are Traceability and Measurement Uncertainty related?

**Yes !**

Measurement Uncertainty is meaningless if  
the Traceability is not known

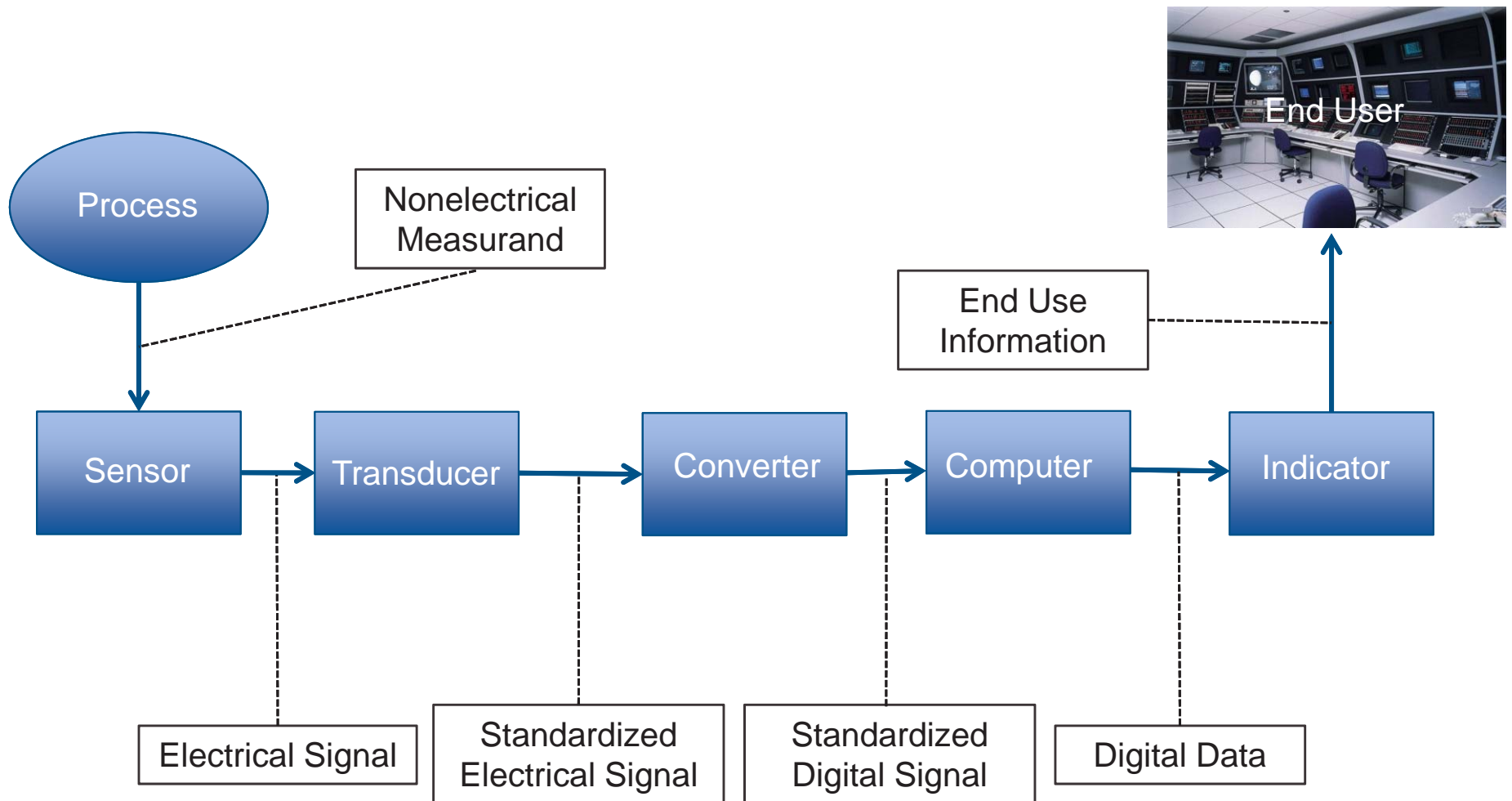
# Traceability Chain



The National Research Council (NRC) is responsible for the investigation and determination of standards and methods of measurement in Canada.

# Sources of Measurement Error

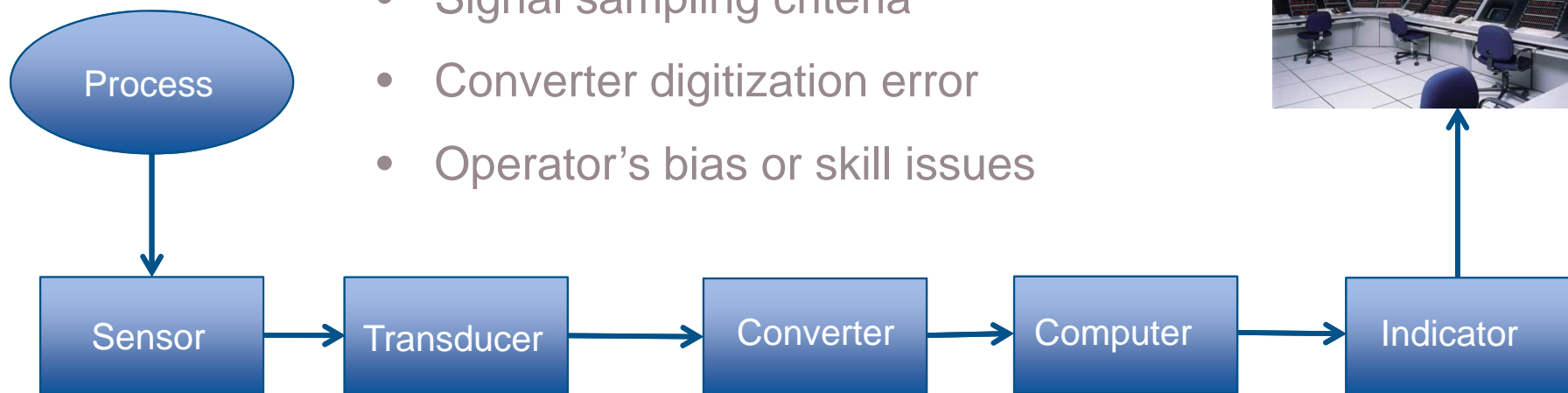
# The Measurement Process



# Sources of Measurement Errors

Industrial measurements are rarely made under perfect conditions, errors and uncertainties may be caused by:

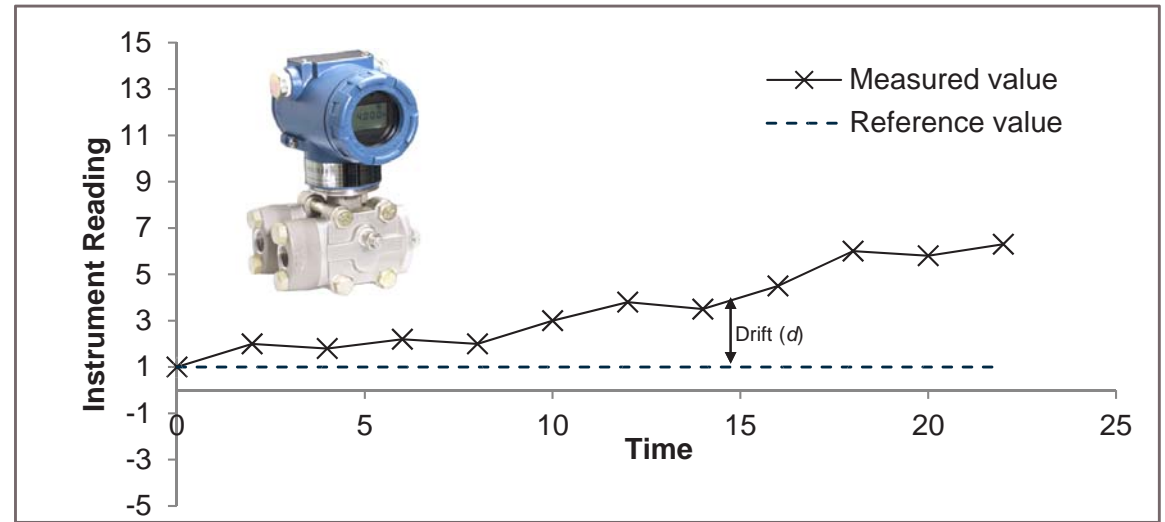
- Stability of the process
- Calibration of sensors
- Noise or drift affecting the electronics
- Environmental conditions
- Signal sampling criteria
- Converter digitization error
- Operator's bias or skill issues



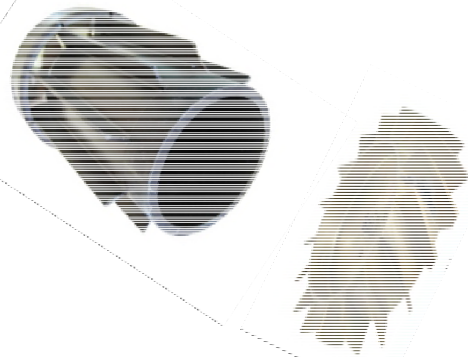
# Sensor Errors



Calibration Error



Long Term Stability

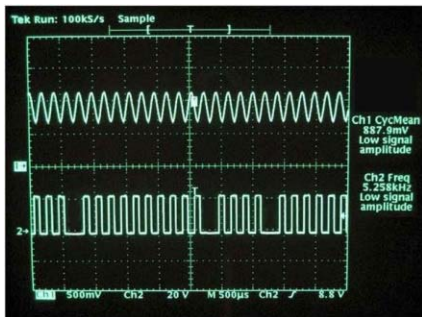


Condition of instruments

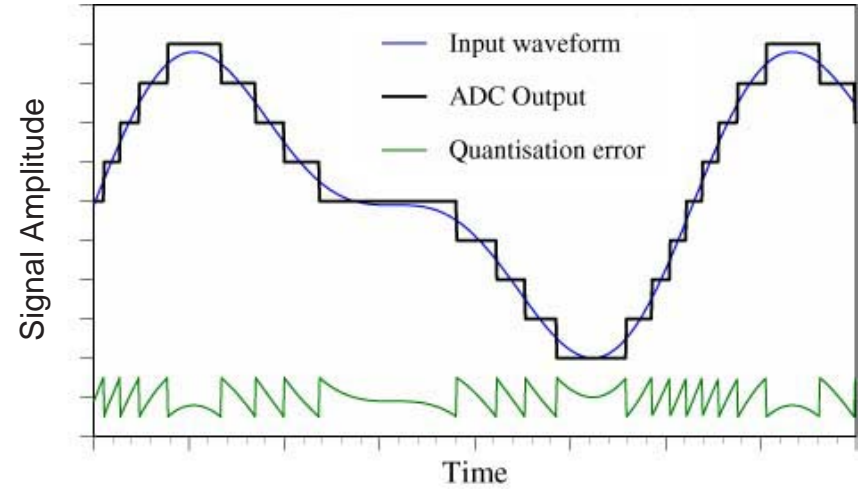




# Digital Measurement Errors



Pulse Detection Error

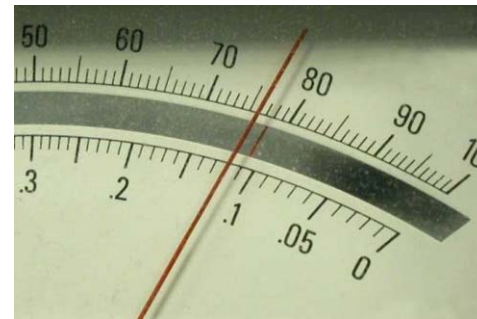
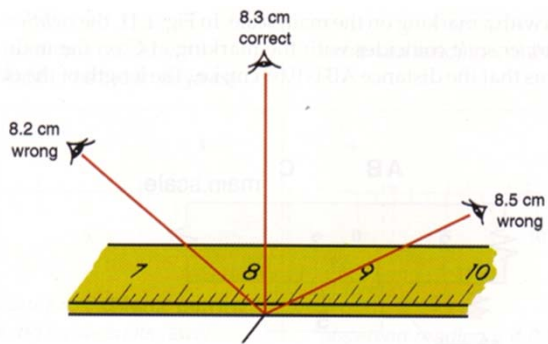


Digital quantisation error

# Observational Errors

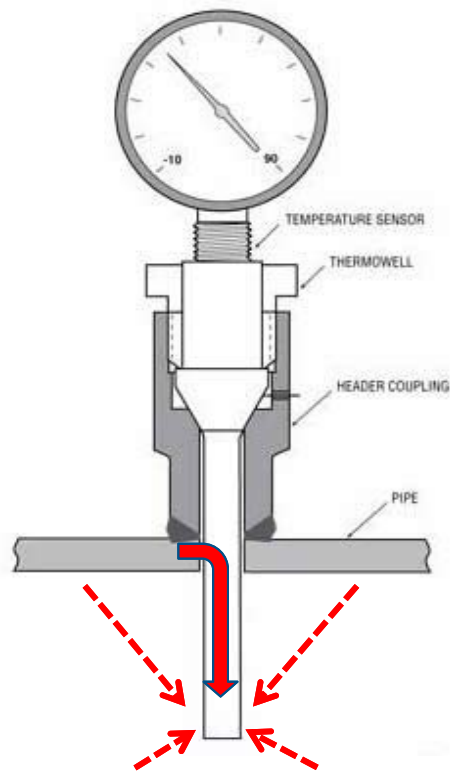


Low resolution instrument

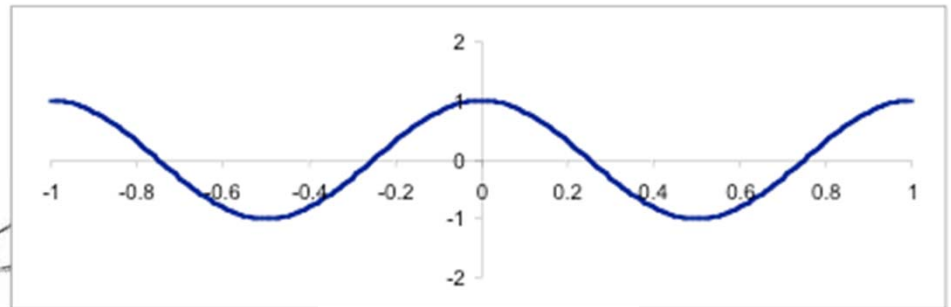
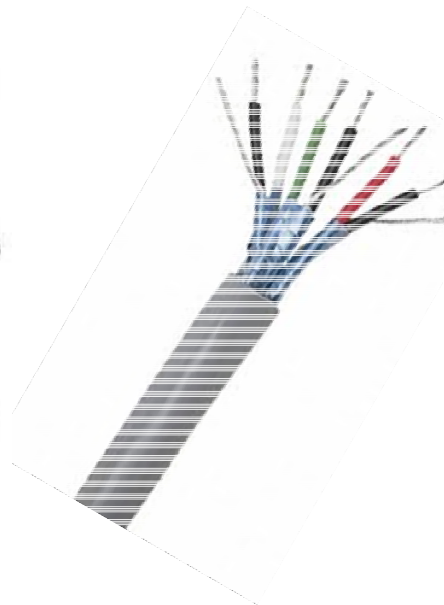


Parallax Error

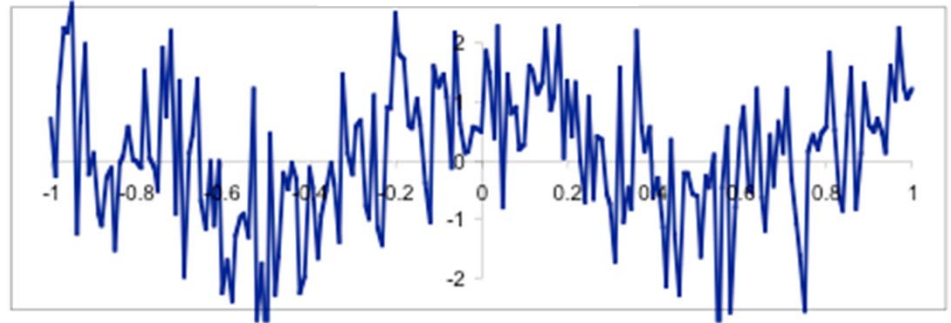
# Errors Introduced by the Environment



Erroneous reading caused by  
conductive and radiative distortion  
of temperature measurement



Process Signal



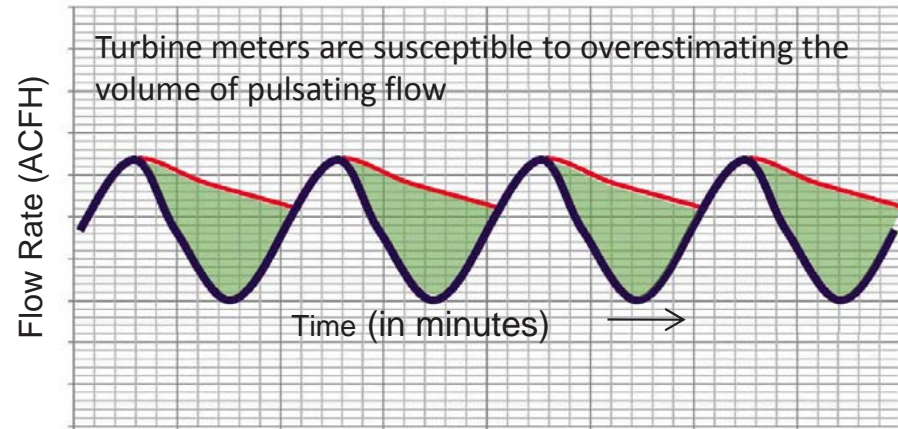
Noisy Process Signal

Noisy signal caused by poor cable  
shielding and/or high electrical  
interference environment

# Installation Related Errors



Some installation problems are obvious



But some installation problems are not so apparent



## .....and don't forget the “*human factor*”

- Operator's personal bias in reading analogue instruments can lead to error and should be accounted for in assessing the uncertainty of the measurement.
- Mistakes made by operators are not considered measurement errors. This type of errors must be eliminated by paying more attention to operator training and making sure that the measurement process is more “error proof”.



# Statistical Treatment of Measurement Data

## Making a Measurement

The readings : 150.0,153.5,156.1,157.8,156.6,155.1,161.8,159.1,156.5,157.0 (psig)

The sum of all readings : 1563.5 psig

The average of 10 readings : 156.4 psig



The best representation of the line pressure is the average (mean) of all data points:

Best estimate of line pressure = 156.4 psig

## Estimating the Spread of the Measurement

In order to quantify the quality of the measurement, we calculate the standard deviation ( $s$ ) of the set of pressure readings:

$$s = \sqrt{\frac{\sum(X - \bar{X})^2}{n - 1}}$$

The *standard deviation* ( $s$ ) of the 10 data points = **3.16 psig**

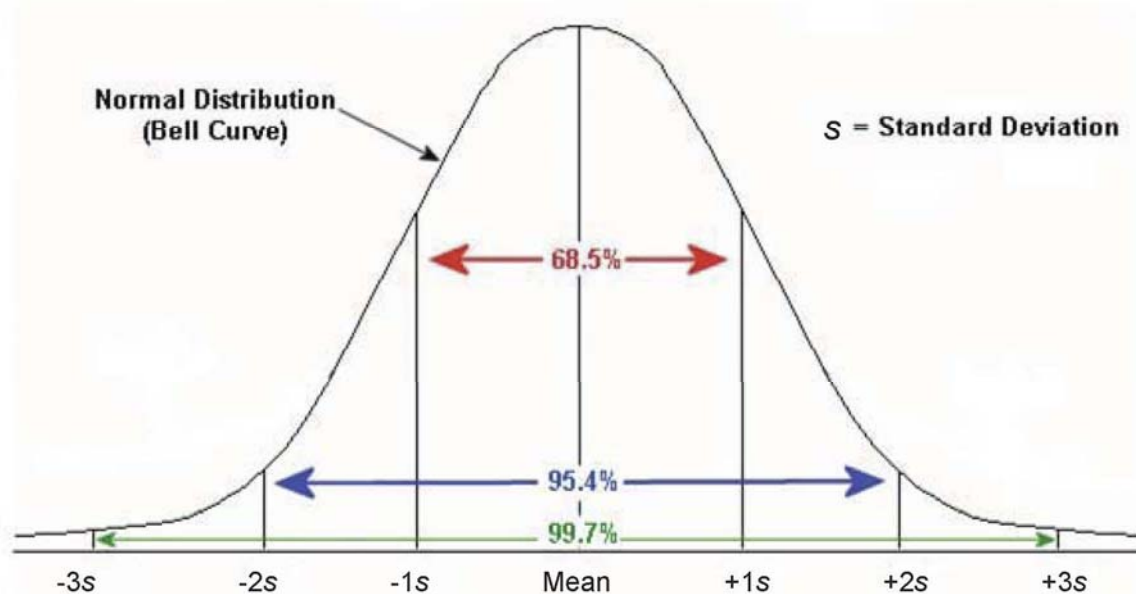


# Interpreting the *Standard Deviation*

## The 70-95-100 Rule

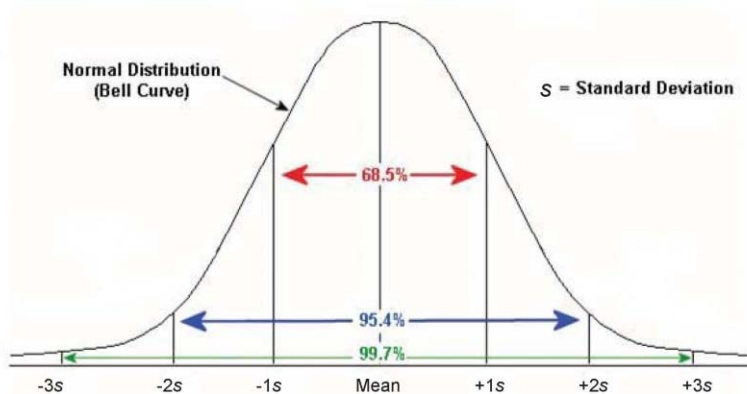
For any data set with *Normal* (Gaussian) distribution:

- about 70% of observations lie within 1 s of the mean
- about 95% of observations lie within 2 s of the mean
- almost all observations lie within 3 s of the mean



# Level of Confidence

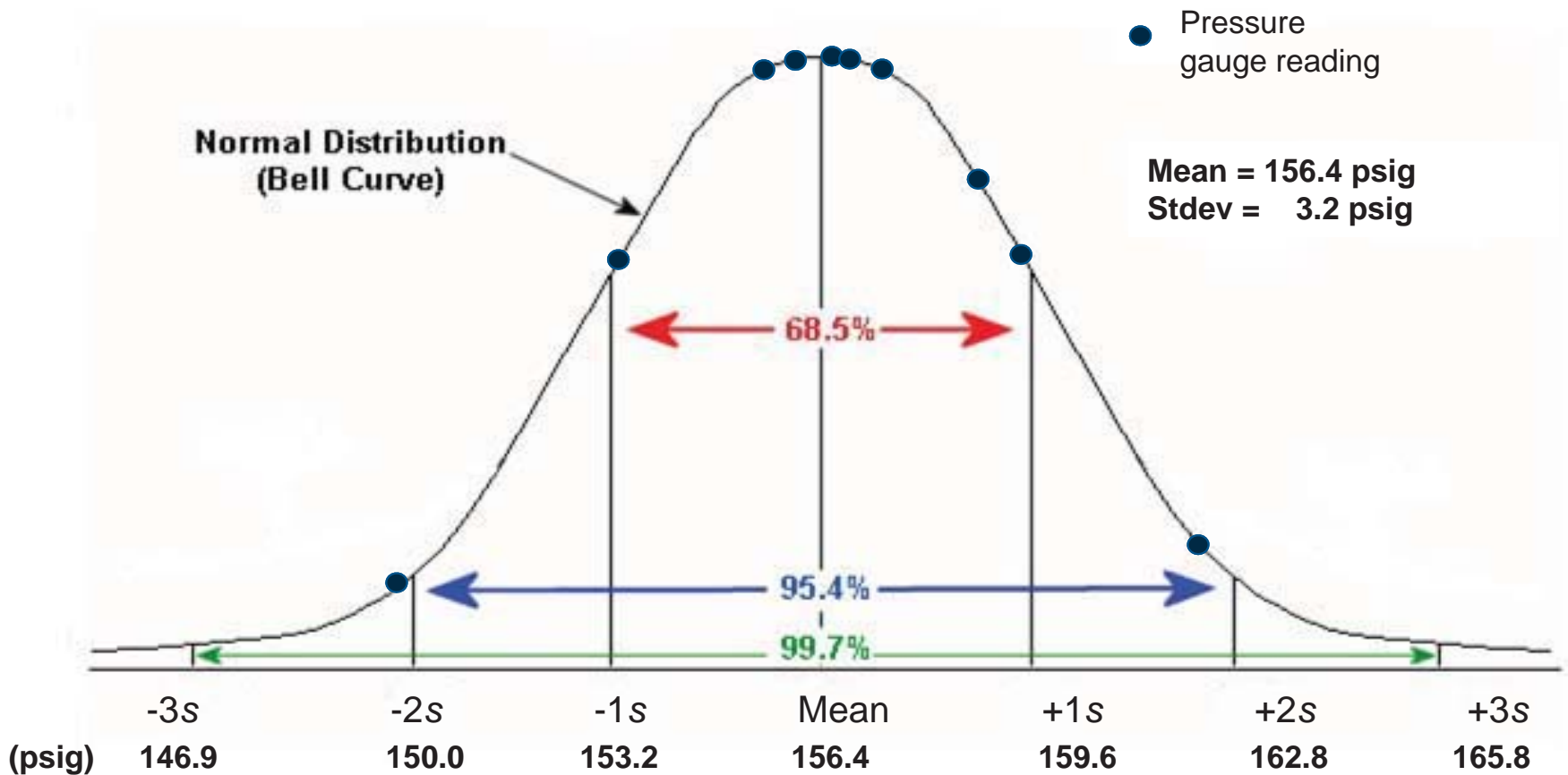
- The expanded uncertainty is calculated by multiplying the standard uncertainty by the coverage factor
- Most calibration institutions report uncertainty at 95% confidence level or  $k=2$



Coverage Factor, k	Level of Confidence
0.68	50.0%
1.00	68.3%
1.65	90.0%
1.96	95.0%
2.00	95.5%
2.58	99.0%
3.00	99.7%

# Applying Standard Deviation to Our Experiment

$$\begin{aligned}\text{Standard uncertainty } (u_s) &= s \\ &= \pm 3.2 \text{ psig}\end{aligned}$$



# Expression of Measurement Uncertainty

From our pressure measurement example

Mean of sample readings = 156.4 psig  
Std. Uncertainty ( $u_s$ ) = 3.2 psig

Most calibration institutes report uncertainty at 95% confidence level, or  $k=2$

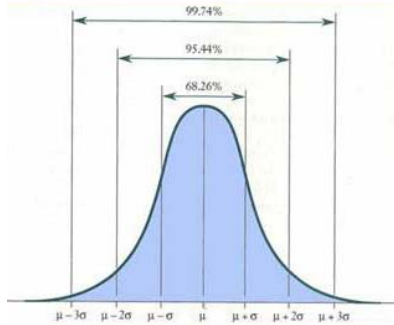
Mean of sample readings = 156.4 psig  
Expanded Deviation ( $U_c$ ) = 6.4 psig

The result of the measurement shall be reported as

*156.4 psig ± 6.4 psig, k=2*

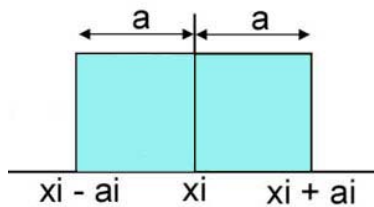
Coverage Factor, k	Level of Confidence
0.68	50.0%
1.00	68.3%
1.65	90.0%
1.96	95.0%
2.00	95.5%
2.58	99.0%
3.00	99.7%

# Other Forms of Probability Distribution



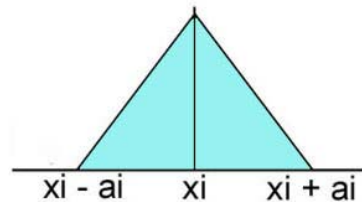
$$u(x_i) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x_i - \mu)^2}{2\sigma^2}}$$

- When type A-evaluations can be shown to follow a normal distribution.



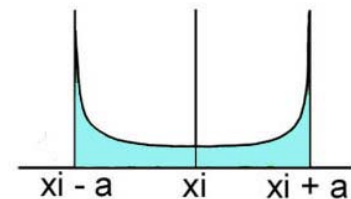
$$u(x_i) = \frac{1}{2a}$$

- When only the variation limits are known.



$$u(x_i) = \frac{3}{4a} \left(1 - \frac{|x_i - \mu|}{a}\right)$$

- When the relationship between variables is known but data is scarce. (sometimes called a “lack of knowledge” distribution).



$$u(x_i) = \frac{1}{2a} \left(1 + \frac{|x_i - \mu|}{a}\right)$$

- When the distribution is bounded and extremes are often realized.

## Standard Deviation of the Mean

For a *Type A* uncertainty evaluation, the estimated standard uncertainty,  $u$ , of the mean of a set of repeated measurements can be calculated as follow:

$$\text{Standard uncertainty of the mean } u_m = \frac{s}{\sqrt{n}}$$

Where

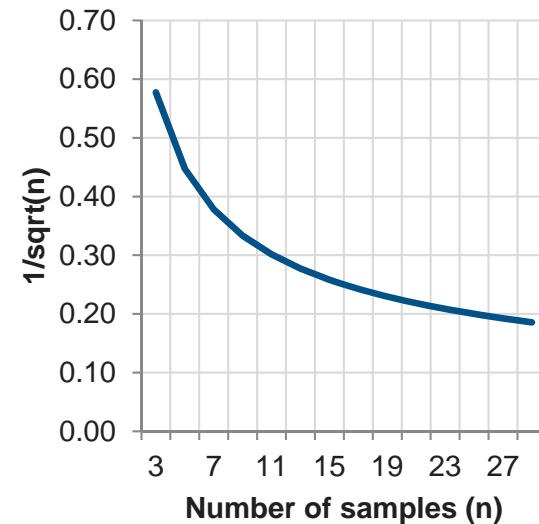
- $s$  - standard deviation of the measurement samples
- $n$  - number of measurement samples

Note that the standard uncertainty ( $u_m$ ) decreases as the number of measurement samples ( $n$ ) increases

# Standard Deviation of the Mean

One cannot expect getting improvement in the standard uncertainty of the mean ( $u_m$ ) indefinitely by repeating the measurement.

The measurement sampling scheme will come to a point of diminishing return quickly once the sample size  $n > 5$ . Very little is to be gained in terms of the standard uncertainty of the mean ( $u_m$ ) if the standard uncertainty of the samples remains the same.



## Combining Standard Uncertainties

Individual standard uncertainties  $u_1 \dots u_i$  obtained by either the *Type A* or *Type B* evaluations can be combined by a “root sum of the squares” process. The result is the combined standard uncertainty  $u_c$ :

$$u_c = \sqrt{u_1^2 + u_2^2 + u_3^2 + \dots + u_i^2}$$

This is the simplest form of the combined standard uncertainty calculation. It is used where the result of a measurement is reached by the addition or subtraction of measurement data.



# Calibration Certificate Example

# Typical Turbine Meter Performance Chart

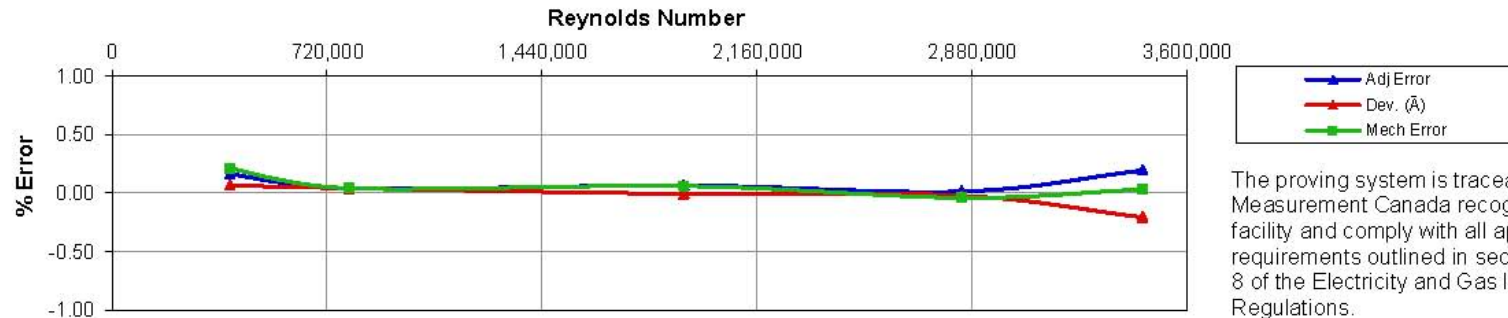
Calibration Chart No. \_\_\_\_\_

<b>Customer</b>	Company A	<b>Test Date</b>	2/12/2014
<b>Customer Address:</b>		<b>Test Medium</b>	CO2
<b>MC Reg Number</b>	40224	<b>Expires</b>	2020
<b>CSO Number</b>	307388	<b>Mech Output (cu.ft./rev)</b>	100
<b>Manufacturer</b>	Sensus	<b>Change Gear</b>	69/49
<b>Meter Model</b>	AAT-18	<b>Mech Output Factor</b>	103.1245
<b>Meter Size (inches)</b>	4	<b>MR Factor (pulses/cu.ft.)</b>	93.8737
<b>Serial Number</b>	13299828	<b>SR Factor (pulses/cu.ft.)</b>	144.1078
<b>Badge Number</b>	P32309	<b>A-Bar (Å)</b>	9.807
<b>Construction</b>	502	<b>Test P (PSIG)</b>	225
<b>P<sub>max</sub> (PSIG)</b>	1440	<b>Test Q<sub>max</sub> (ACFH)</b>	16,227
<b>Q<sub>max</sub> (ACFH)</b>	18,000	<b>Test Q<sub>min</sub> (ACFH)</b>	1,808
<b>Q<sub>min</sub> (ACFH)</b>	1,800	<b>Test Re<sub>max</sub></b>	3,452,392
<b>MR Spin Time Test (sec)</b>	156	<b>Test Re<sub>min</sub></b>	394,351
<b>SR Spin Time Test (sec)</b>	296		

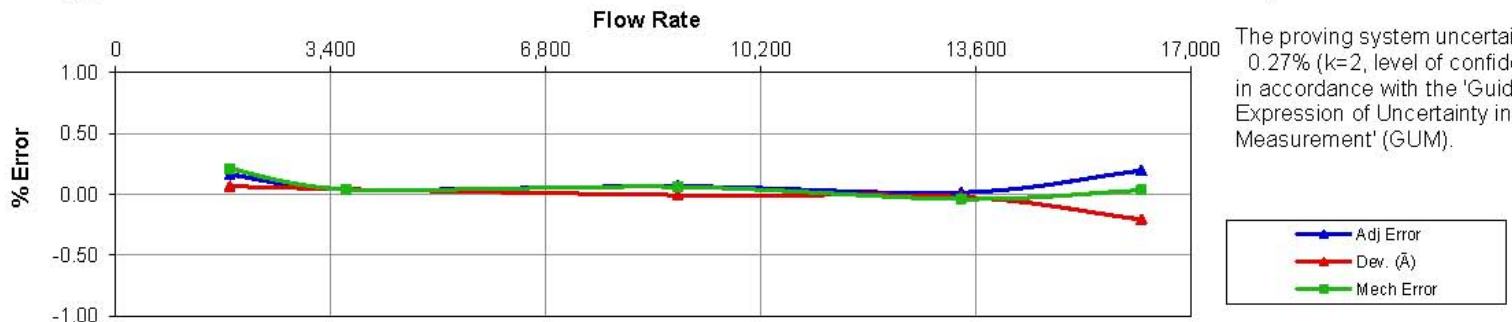
Test Point	Test Point % of Q <sub>max</sub>	Pressure (PSIA)	Temp (°F)	Flow Rate (ACFH)	Re	% Error Adjusted	% Error Mech	Dev. (Å)	CMC (%)	U <sub>tot</sub>
1	90%	240.6	69.0	16,227	3,452,392	0.20	0.03	-0.21	0.27	0.27
2	74%	240.5	70.6	13,375	2,844,534	0.01	-0.04	-0.02	0.27	0.27
3	49%	240.1	69.4	8,894	1,914,838	0.07	0.06	-0.01	0.27	0.27
4	20%	240.1	68.3	3,649	793,898	0.04	0.04	0.04	0.27	0.27
5	10%	240.1	67.9	1,808	394,351	0.16	0.21	0.07	0.27	0.27
6										
7										
8										
9										
10										

Table 1 Meter Calibration Results

# Typical Turbine Meter Performance Chart



The proving system is traceable to a Measurement Canada recognized test facility and comply with all applicable requirements outlined in section 7 and 8 of the Electricity and Gas Inspection Regulations.



The proving system uncertainty is 0.27% (k=2, level of confidence 95%) in accordance with the 'Guide to the Expression of Uncertainty in Measurement' (GUM).

Comments: \_\_\_\_\_

Tested by: R. Guilfoyle  
(Print)

444 Okanagan Avenue East, Penticton B.C., Canada, V2A 3K3  
1-800-667-4338 [www.FortisBC.com/Measurement](http://www.FortisBC.com/Measurement)  
FortisBC Measurement is a department of FortisBC Energy Inc.

Measurement Technician  
(Designation)

Control No.:	QA-176D
Issue Date:	Mar/13
Revision:	4

\_\_\_\_\_  
(Signature)

Reviewed:

# Typical Turbine Meter Performance Chart

## Test Conditions

The meter under test (MUT) was calibrated in a closed loop with the test medium pressurized to the test pressures stated in Table 1. The test results were calculated by comparing normalized flow data with reference to the MUT meter run.

## Environmental Conditions

Temperature:	$23 \pm 5^{\circ}\text{C}$
Pressure:	$14.7 \pm 1 \text{ psia}$
Relative Humidity:	10 to 80%RH

## Traceability Statement:

The Calibration Laboratory Assessment Service (CLAS) of the National Research Council of Canada (NRC) has assessed and certified specific calibration capabilities of this laboratory and traceability to the International System of Units (SI) or to standards acceptable to the CLAS program. This certificate of calibration is issued in accordance with the conditions of certification granted by CLAS and the conditions of accreditation granted by the Standards Council of Canada (SCC). Neither CLAS nor SCC guarantee the accuracy of individual calibrations by accredited laboratories.

# Typical Turbine Meter Performance Chart

The error percent calculation:

$$e \% = \frac{V_i - V_r}{V_r} \times 100\%$$

where

$e$  = metering error  
 $V_i$  = volume indicated by the meter under test  
 $V_r$  = reference volume

For Auto-adjust meters:

$$V_{adj} = V_m - V_s$$

$$V_m = \frac{C_m}{K_m}$$

$$V_s = \frac{C_s}{K_s}$$

where

$V_{adj}$  = adjusted volume  
 $V_m$  = volume registered by main rotor  
 $V_s$  = volume registered by sensing rotor  
 $C_m$  = volume count registered by main rotor  
 $K_m$  = meter factor for main rotor  
 $C_s$  = volume count registered by sensing rotor  
 $K_s$  = meter factor for sensing rotor

and

$$\Delta A = \left[ \frac{V_s}{V_m - V_s} \times 100 \right] - \bar{A}$$

where

$\bar{A}$  = average value of the factory sensing rotor % adjustment  
 $\Delta A$  = % deviation in field operation from factory calibration

## Reference Meters:

Description	Meter ID
8 " Run 100	10402409
8 " Run 200	10402408
4 " Run 300	10506088
8 " Run 400	10402410
8 " Run 500	10402407
2" Run 600	99967

## In Summary

- What is Measurement
- “True” Value, Error, Accuracy, Precision, and Uncertainty
- Sources of Data Error in Measurement
- Statistical Treatment of Measurement Errors
- Calibration Certificate Example
- Questions

*Thank You !*

Contact information:

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