Measurement Uncertainty Analysis of a Closed Loop High Pressure Turbine Meter Calibration Facility

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Introduction

Understanding the principles of uncertainty analysis is essential in gauging the quality of measurement results. In designing a turbine meter calibration facility, the quality of calibration, or in other words the calibration uncertainty of the facility, is influenced by the designer's detailed and exact knowledge of all the physical processes used to derive the final calibration result. In this paper, the author explains the methodology used to develop the mathematical model for estimating the calibration uncertainty of the Triple Point High Pressure Meter Calibration Facility located in Penticton, BC. The measurement uncertainty contribution by each measurement component is examined. The effects of flow Reynolds number, composition of the test medium, physical configuration of the meter run, and the stability of temperature and pressure control of the test loop, are explored. Finally, the various methods of expressing measurement uncertainty are discussed.

Test Loop Configuration

Triple Point is a high pressure closed-loop turbine meter proving facility. While the test loop may be used to prove turbine meters using many different gas media at low pressure when active loop cooling is not required, the Triple Point loop cooling system is specifically design to operate with liquid carbon dioxide as a refrigerant. The meter proving loop is essentially configured as a transfer prover system. Figure 1 illustrates the general layout of the Triple Point turbine meter proving facility.

The primary parameters measured in the test loop are temperature, pressure, and flow. High precision RTDs are used to monitor temperatures downstream from each of the turbine meters. In order to minimize the time lag in temperature measurement, these RTDs are installed without thermowells. This allows the RTDs to respond quickly to temperature changes. Temperatures are measured and converted into the proper engineering unit scale by the data acquisition modules in a programmable logic controller (PLC). Pressure differences between the master turbine meters (MR) and the meter under test (MUT) are measured by differential pressure transducers in inches water column ("w.c.). Differential pressure measurements are used instead of static pressure measurements in order to improve the measurement resolution and stability. A novel "floating" static pressure reference system provides a stable reference point for each of the differential pressure transducers.

The Triple Point reference meter bank consists of four 8-inch and one 4-inch Instromet XIC series turbine meters. Each one of the reference meters is equipped with an integral inlet flow conditioner as well as a 19-tube bundle flow straightener upstream in the meter run. In order to meet the full operating Reynold's number range of Triple Point, the master meters were calibrated at three different NMI (Nederlands Meetinstituut) facilities in Europe. The low Reynold's number air flow tests were performed at NMI's Silvolde facility; while the medium and high Reynold's number flow tests were conducted at Utrecht and Bergum in the Netherlands.

The mid-point K-factor of each reference meter is provided by the manufacturer. Calibrated metering errors were expressed as a percentage volume deviation from the true volume established by a recognized standard. The relationship between turbine meter error and the flow condition as defined by its Reynold's number is well understood and is supported by organizations such as Measurement Canada, AGA [1] and CEN [2]. For each reference meter, a polynomial equation was developed to characterize it for the full Triple Point operating range based on the NMI calibration data.

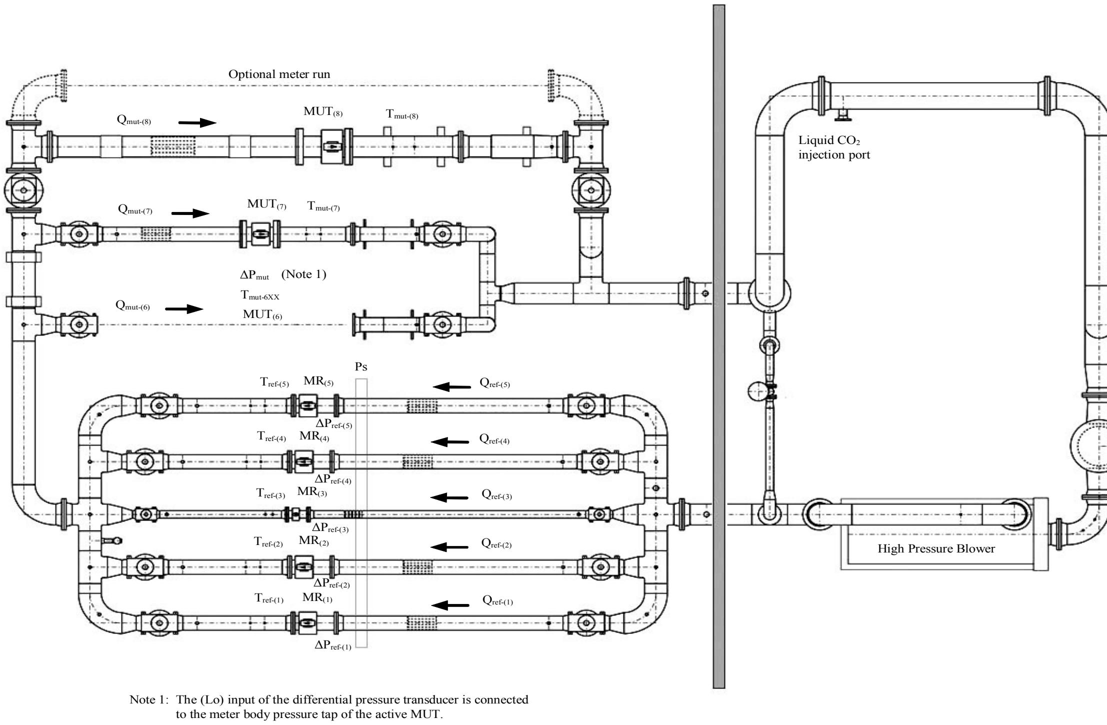

Figure 1 Triple Point test loop general layout

Combination of more than one reference meter may be engaged in a calibration run in order to accommodate flow rates up to 230,000 ACFH. The test loop is capable of generating maximum Reynold's number of approximately 9,200,000. Only one of the three meter-under-test runs, (MUT-6 to MUT-8), may be engaged for each meter calibration test. The unused MUT runs are disabled and secured following a safety lock-down procedure. Special flange alignment tools are used to properly mount a meter-under-test in order to eliminate potential errors caused by piping misalignment. A purging procedure is used to flush out remnant air contaminants from the loop to a level compatible with the purity of carbon dioxide from the storage tank (better than 99.9%). Both theoretical analysis and field tests confirm that contaminant at this level would not cause any noticeable (<0.01%) shift in the meter calibration results. Test gas leakage from the system sufficient to cause calibration error is easily detectible either by the ambient CO2 level monitor in the test area, or by following the test procedure of checking the flange joints using a portable CO2 detector each time after a new meter-under-test is mounted.

Modeling Equations

The design of a transfer prover system is based on the principle of conservation of mass in a test loop. In general, the mass of gas molecules passing through the master meter must equal to the mass of the gas molecules passing through the meter-under-test. Expressing this relationship in the following equations:

$$\dot{M}_{REF} = \dot{M}_{MUT} \tag{1}$$

$$\frac{M_{REF}}{t} = \frac{M_{MUT}}{t} \tag{2}$$

$$\frac{\rho_{REF} \times V_{REF}}{t} = \frac{\rho_{MUT} \times V_{MUT}}{t} \tag{3}$$

where $\dot{M}_{REF}$ and $\dot{M}_{MUT}$ represent mass flow, t represents time, ρ represents density, and V represents volume. From the *equations of state* which describe the behavior of gases under temperature (T) and pressure (P), we have the following relationship:

$$\rho = \frac{P}{ZRT} \quad (4)$$

in which R is the universal gas constant, and Z is the compressibility factor of the gas. Substituting (4) into (3), we derive a flow rate equation (5) which describes the flow condition of the reference meter and the meter-under-test in terms of the physical state of the gas:

$$Q_{REF} \times \frac{P_{REF}}{Z_{REF} \times R \times T_{REF}} = Q_{MUT} \times \frac{P_{MUT}}{Z_{MUT} \times R \times T_{MUT}} \quad (5)$$

Rearranging equation (5), we now have an expression that describes the flow rate Q_{MUT} at the meter-under-test, expressed in terms of the flow rate at the reference meter Q_{REF}:

$$Q_{MUT} = Q_{REF} \times \frac{P_{REF} \times T_{MUT} \times Z_{MUT}}{P_{MUT} \times T_{REF} \times Z_{REF}} \quad (6)$$

The gas volume flow at a reference meter during a period of time t is represented by:

$$V_{REF} = \frac{N_{REF}}{K_{REF}} \times \frac{100 - E_{REF}}{100} \quad (7)$$

in which N_{ref} is the number of pulses produced by the reference meter, while E_{ref} is metering error, expressed in % deviation of the indicated volume from the true volume passing through the reference meter.

Expression (7) describes the volume of gas flowing through the reference meter (for the moment, let us assume only one reference meter run is engaged) during time period t. The quantity of gas passing through the meter-under-test in the same time period is represented by combining (6) and (7) :

$$V_{MUT} = \frac{N_{REF}}{K_{REF}} \times \frac{(100 - E_{REF})}{100} \times \frac{P_{REF} \times T_{MUT} \times Z_{MUT}}{P_{MUT} \times T_{REF} \times Z_{REF}} \quad (8)$$

To simplify (8), we create four new correction factors and define them as follow:

a. Reynold's number correction factor

$$C_{RE} = \frac{(100 - E_{REF})}{100} \quad (9)$$

E_{REF} in equation (14) is the error of the reference meter operating at the same Reynold's number (Re) as the test stream. E_{REF} is determined by an nth order polynomial function of the variable Re which defines the calibration characteristic of the meter. The Reynold's number (Re) is a dimensionless number defined by:

$$\mathrm{Re} = \frac{\rho \upsilon D}{\eta} \quad (10)$$

in which ρ is density, v is velocity, D is the pipe diameter, and η is the dynamic viscosity of the gas.

b. Temperature correction factor

$$C_T = \frac{T_{MUT}}{T_{REF}} = \frac{T_{MUT}(°F) + 459.67}{T_{REF}(°F) + 459.67} \quad (11)$$

The temperature correction factor C_T is a ratio of the meter-under-test temperature versus the reference meter temperature, both expressed in an absolute temperature scale. This factor compensates for the gas volume changes due to thermal expansion.

c. Pressure correction factor

$$C_P = \frac{P_{REF}}{P_{MUT}} = \frac{Ps(psia) \times 27.68067 + \Delta P_{REF}(\text{"}w.c.)}{Ps(psia) \times 27.68067 - \Delta P_{MUT}(\text{"}w.c.)} \quad (12)$$

The pressure correction factor C_P compensates for gas volume changes due to gas compression.

d. Compressibility correction factor

$$C_Z = \frac{Z_{MUT}}{Z_{REF}} \quad (13)$$

The compressibility correction factor C_Z compensates for gas volume changes caused by changes in gas compressibility factor.

These four correction factors allow us to take the amount of gas registered by volume at the reference meter and convert it to match the flow conditions of the meter-under-test. Equation (8) can now be simplified into:

$$V_{MUT} = \frac{N_{REF}}{K_{REF}} \times C_{RE} \times C_P \times C_T \times C_Z \qquad (14)$$

The measurement error of the meter-under-test is represented by:

$$e_{MUT} = \left(\frac{V_{MUT(observed)}}{V_{MUT(true)}} - 1 \right) \times 100\% \qquad (15)$$

Since the true volume of the gas passing through the meter-under-test is the same quantity of gas passing through the reference meter, but converted to the meter-under-test conditions, equation (15) can be rewritten as:

$$e_{MUT} = \left(\frac{\dfrac{N_{MUT}}{K_{MUT}}}{\dfrac{N_{REF}}{K_{REF}} \times C_{RE} \times C_P \times C_T \times C_Z} - 1 \right) \times 100\% \qquad (16)$$

For high flow rate calibration runs in which more than one 8-inch reference meter is engaged, equation (16) becomes:

$$e_{MUT} = \left(\frac{\dfrac{N_{MUT}}{K_{MUT}}}{\sum_{i+1}^{I} \dfrac{N_{REF(i)}}{K_{REF(i)}} \times C_{RE(i)} \times C_{P(i)} \times C_{T(i)} \times C_{Z(i)}} - 1 \right) \times 100\% \qquad (17)$$

in which I is the number of reference meters used in the calibration test. In equation (17), the sum of all of the corrected reference meter flows is used to compare against the flow registered by the meter-under-test to produce the metering error figure e_{MUT}.

Computation of Metering Errors

Measurement data from the Triple Point test loop is collected by a PLC based data acquisition system in the test area, and passed to a LabView based application software residing in a control room PC. The PLC system also performs primary engineering unit conversions. The LabView software further processes the data for the HMI (human machine interface) displays and the data archival functions.

The test data analysis engine of the Triple Point system is a Microsoft Excel spreadsheet program. Data are entered into the appropriate cells after each test run by executing a set of Macro instructions. The gas *equations of state* calculations on the spreadsheet were handled by the NIST (National Institute of Standards and Technology) *REFPROP* (*REF*erence fluid *PROP*erties) Version 7.0. The physical properties of the carbon dioxide or any other test gases in the test loop were calculated using the *REFPROP DDL* module. The calculated results, including density, compressibility, and viscosity, were returned to the spreadsheet to generate Reynold's number *Re*, and gas compressibility factor *Z*. These variables were then used to compute the correction factors C_T, C_P, and C_Z by applying equation (11), (12), and (13). The error figure of the meter-under-test, e_{MUT} was calculated by using equation (17).

Sources of Uncertainty

We will identify the various factors that cause measurement uncertainty in e_{MUT} in this section. This process generally follows the procedure described in the Measurement Canada document "Recommendations for the Determination of Measurement Uncertainty in Simple Two Meter Comparisons" [3], and the guidelines published in the ISO "Guide To The Expression of Uncertainty In Measurement" (GUM) report [4].

The GUM report classifies the evaluation of measurement uncertainty into Type A and Type B. A Type A evaluation is built on and calculated from a series of repeated observations of a measurement process. For a new facility such as Triple Point for which repeated observations and long term measurement data are still being collected, we rely on the Type B evaluation approach. A Type B evaluation is based on using available knowledge of the process and process elements. The Type B contributors are those that must be determined by non-statistical methods. Table 1 shows a summary of the uncertainty characteristics of these elements in Triple Point.

Uncertainty in reference meters

Each of the Triple Point reference meters was calibrated at NMI and has a current NMI calibration certificate specifying the measurement uncertainty figure with a coverage factor of k = 1.96. Based on normal distribution, these uncertainty figures correspond to a confidence level of approximately 95%. The multiple reference meter configuration of Triple Point offers more operating flexibility by allowing the selection of the best single or combination of reference meter(s) for optimal calibration uncertainty.

Variable Name	Factors Contributing to the Uncertainty of Variable	Standard Uncertainty (u_c)	Sensitivity Coefficient ($\partial f / \partial x_i$)
N_{MUT}	MUT Pulse Counting Pulse counting uncertainty Other additional assumptions	$N_{MUT} \pm 1$ count	For MUT Count $\geq$ 10,000 $\frac{\partial f}{\partial N_{MUT}} = 0.01\% / pulse$
N_{REF}	Reference Meter Counting Pulse counting uncertainty Other additional assumptions	$N_{REF} \pm 1$ count	For Reference Meter count $\geq$ 10,000 $\frac{\partial f}{\partial N_{REF}} = -0.01\% / pulse$
K_{REF}	Reference Meter Calibration uncertainty CO_2 equivalency uncertainty Temperature sensitivity Pressure sensitivity Sensitivity to gas composition	$K_{REF} \pm$ 0.13% or $K_{REF} \pm$ 0.0598 pulse/ft^3	$\frac{\partial f}{\partial K_{REF}} = 2.15\% / pulse / ft^3$
T_{MUT}	Temperature – MUT Calibration uncertainty Sampling errors Resolution errors Other additional assumptions	$T_{MUT} \pm$ 0.09ºF	$\frac{\partial f}{\partial T_{MUT}} = -0.22\% / ^o F$
ΔP_{MUT}	Differential Pressure – MUT Calibration uncertainty Sampling errors Resolution errors Other additional assumptions	$\Delta P_{MUT} \pm$ 0.036" w.c.	$\frac{\partial f}{\partial \Delta P_{MUT}} = 0.04\% / '' w.c.$
P_S	Static Pressure Calibration uncertainty Sampling errors Resolution errors Other additional assumptions	$P_S \pm$ 0.036 psia	$\frac{\partial f}{\partial P_S} << 0.001\% / psi$
T_{REF}	Temperature – Reference Meters Calibration uncertainty Sampling errors Resolution errors Other additional assumptions	*TREF ± 0.09ºF*	$\frac{\partial f}{\partial T_{REF}} = 0.22\% / ^o F$
ΔP_{REF}	Differential Pressure – Reference Meters Calibration uncertainty Sampling errors Resolution errors Other additional assumptions	$\Delta P_{REF} \pm$ *0.036"w.c.*	$\frac{\partial f}{\partial \Delta P_{REF}} = -0.04\% / '' w.c.$
u_{Rio}	MUT put-in and take-out reproducibility		1

Table 1 Triple Point Uncertainty Budget

Several factors affect the overall measurement uncertainty of each reference meter. The combined uncertainty of a reference meter, $u_c(M)$, may be broken down into the following components:

$$u_c(M) = \sqrt{u^2_{cal(M)} + u^2_{flow\_sensitivity(M)} + u^2_{tp\_sensitivity(M)}} \tag{18}$$

where $u^2_{cal(M)}$ is the calibration uncertainty of the reference meter, $u^2_{flow\_sensitivity(M)}$ is the reference meter measurement uncertainty caused by flow rate sensitivity, and $u^2_{tp\ sensitivity(M)}$ is the temperature and pressure sensitivity of the reference meter. Since the focus of this paper is to demonstrate the methodology of estimating measurement uncertainty, we would simplify the discussion by choosing the worse case metering deviation uncertainty figure of ±0.21% (coverage factor 1.96) from the NMI calibration certificates for our calculation. This number was converted to the standard uncertainty figure of ±0.11% (coverage factor = 1). The uncertainty figures at flow rates outside of Triple Point's operating range were not included. Further discussion of the second and the third terms in (18) will be provided in subsequent sections.

Uncertainty of the equivalency of CO_2

In the natural gas industry, it is a common practice to obtain the calibration of a turbine meter in atmospheric air and apply the same calibration curves for natural gas applications. There are general concerns about the validity of such an approach mainly because of the mismatch in gas density and Reynold's number of the test medium. There is good theoretical understanding and support in the industry regarding the relationship between metering error and Reynold's number, although not very much turbine meter test data for carbon dioxide gas is available for consideration at this time. Due to the lack of carbon dioxide test data, Terasen Measurement contracted Southwest Research Institute in 2003 to perform turbine meter calibration in both carbon dioxide and natural gas at various operating pressures in order to experimentally establish the metering error versus Reynold's number relationship. The result of this study was published in a report entitled "Dual Fluid Test Program for Turbine Meter Calibrations" [5]. The report concluded that:

"*In summary, the calibration of these turbine meters in carbon dioxide agreed with calibrations in natural gas at the same densities and Reynolds numbers, generally to within 0.15%. It can be concluded that turbine meters calibrated in carbon dioxide can be used in natural gas applications. Measurement errors associated strictly with the difference in calibration factor between a meter calibrated in carbon dioxide and the same meter calibrated in natural gas would be well within the maximum uncertainty allowed by American Gas Association Report No. 7, Measurement of Natural Gas by Turbine Meters, provided that densities and Reynolds number were matched between calibration and field conditions.*"

Due to the absence of further experimental data on CO_2 tests at this time, we will make a worse case assumption that the 0.15% uncertainty observed at SwRI was entirely caused by the difference in the fluid properties of carbon dioxide and natural gas. Modifying equation (18) to reflect this assumption, we have:

$$u_c(M) = \sqrt{u^2_{cal(M)} + u^2_{flow\_sensitivity(M)} + u^2_{tp\_sensitivity(M)} + u^2_{CO_2(M)}} \qquad (19)$$

In this case, the new combined uncertainty of a reference meter, $u_c(M)$, is equal to the root-sum-square of the combined uncertainty of the meter established in (18), and the uncertainty contribution due to the carbon dioxide test medium.

It should be noted that the approach taken above would produce a very conservative value for $u_c(M)$. It could be argued that metering error caused by the difference between the two fluids is systematic in nature with respect to Reynold's number. If the operating principle of the meter under test is the same as the reference meters, one may conclude that the fluid property difference would affect all of these meters the same way, therefore negating the effect of any systematic bias. Accumulation of future test results from Triple Point would allow us to substantiate this argument.

Temperature sensitivity of reference meters

Although the effect of temperature on the gas test medium is well known and understood, there is little data available concerning how the calibration of a turbine meter may be affected by changes in its body temperature. This item is identified as a category (d) contributor of measurement uncertainty in GUM. The Triple Point test facility has complete control on the operating temperature of the test loop. For the narrow test temperature range the Triple Point reference meters are exposed to, the calibrations of these meters are assumed to be unaffected.

Pressure sensitivity of reference meters

It has been known that the calibration of turbine meters is affected by their operating pressures. Many papers have been written about the effect of Reynold's number on the calibration of turbine meters. As noted in equation (4) and (10), Reynold's number is a function of gas density. Gas density is directly proportional to the operating pressure. At a high enough Reynold's number, a turbine meter behaves strictly as a "Reynold's number machine". Triple Point is a meter testing facility design specifically to address the *K*-factor versus Reynold's number interdependency. In order to evaluate the Reynold's number sensitivity of the reference meters, all of the reference meters were calibrated at three different NMI facilities in order to characterize them over a wide operating pressure range. The measurement errors of these reference meters obtained from the NMI calibration charts were converted into a set of *n*th order polynomial functions of Reynold's number, thus fully accounting for the pressure effect on them.

Equations of state and transport properties variable

Several gas thermodynamic and transport properties, namely, density (ρ), compressibility (Z), and viscosity (η), are used to derive the Reynold's numbers in (10), and the correction factors C_Z in (13). Uncertainty caused by these types of variables are identified as category (h) in the GUM document, which described the inexact values of constants and other parameters obtained from external sources and used in the data reduction calculation. Triple Point makes use of the *REFPROP* database *DDL* module developed by NIST to generate figures for these properties. At pressures up to 30 MPa (4,350 psia) and temperatures up to 523 K (250 ºC), the estimated uncertainty in *REFPROP*'s carbon dioxide density ranges from 0.03% to 0.05%. For a much narrower operating pressure and temperature range at the Triple Point facility, the estimated uncertainty is expected to be much smaller than 0.03%. The estimated uncertainty in viscosity of carbon dioxide is 0.3% for the same operating range.

The compressibility correction factor C_Z can be expressed as ratio of gas densities by combining equation (4) and (13). Since both density figures are derived by the same mathematical model with the same source of errors, the overall uncertainty of C_Z would be further reduced due to the favorable interdependency of these two density variables in the deriving formula and is therefore considered negligible.

Reynold's number is derived as a quotient of density (ρ) and viscosity (η) in equation (10). Since the estimated uncertainty of viscosity is an order of magnitude larger than that of density, it can be assumed that the uncertainty contribution of viscosity dominates the Reynold's number calculation, and the effect of density is negligible.

Calibration of temperature and pressure sensors

Nearly all of the variables in equation (17) for deriving metering error e_{mut}, with the exception of meter pulse counts and the *K*-factors, are either temperature and pressure measurements, or a mathematical function of the temperature or pressure variables. The uncertainty in the calibration of sensors $u_{calibration(C)}$ is identified in the GUM document as category (g) for inexact values of measurement standards and reference materials. These values are generally taken directly from the calibration certificates of the instruments.

The measurement uncertainty specification of the Rosemount differential pressure transducers used at Triple Point was given by the manufacturer as ±0.025% of span (250"w.c.) with k=3. Assuming normal distribution, this would translate into a standard uncertainty figure of ±0.021"w.c.. Similarly, the standard measurement uncertainties of the static pressure transducer and the RTDs would be expressed as ±0.021 psia and ±0.09 ºF respectively.

Pulse counting uncertainty

The Triple Point pulse counting process is gated by the internal clock of the PLC data acquisition system. The comparison of gas flow through the reference meter and the meter-under-test is based on measuring the volume of gas passing through each device for the same period of time *t*. The absolute value of the time period *t* is not important, but the time period for measuring flow volume through the two devices must be exactly the same. That is to say, the pulse counting process at the reference meter and the meter-under-test must start and stop at the same time. Since the gating signals responsible for starting and stopping the counts in both devices are the same and go through the same electronic circuitries, the worst case maximum timing discrepancy between the reference meter counts and the meter-under-test counts is expected to be less than ± 5 msec according to the specification of the data acquisition system. Over a minimum meter test time of 60 seconds, the timing error is estimated to be less than ± 0.008%.

Connecting volume consideration

Triple Point test procedures take good care of ensuring the integrity of the gas volume between the reference meter and the meter-under-test. Since the test loop normally operates at relatively high pressure (compared to an atmospheric prover), an external leak in the piping system is very noticeable in terms of the pressure drop caused by the leak. Small leaks can be detected by the ambient CO2 detectors in the test area as well as by a portable CO2 detector used by the operator to check the flange couplings after a new meter-under-test is mounted. Metering error caused by external leakage is negligible when the proper operating procedure is observed.

Leakage through the isolation ball valves at the meter runs may also introduce metering error. Since this type of leakage is driven entirely by the differential pressure across the isolation valves, it can be eliminated by equalizing the pressure around the entire loop during a test.

Since both the reference meters and the meter-under-test are in close proximity and exposed to the same environment, it is assumed that the contribution of dimensional changes in the meter runs due to temperature or pressure differences is very small and may be considered negligible.

Uncertainty caused by contaminants in the test stream

The carbon dioxide gas used in the Triple Point test loop is a food grade product with purity better than 99.9% upon delivery, as a liquid, to the storage tank. The only source of contaminant in the loop is atmospheric air introduced during the mounting of a new meter-under-test. A boot-strapping purge procedure used by the test operator before testing a new meter ensures that the air contaminant is keep below 1,000 ppm. The level of air contaminant in the test loop is monitored by an on-line oxygen level sensor.

The presence of air contaminant affects the physical properties of the carbon dioxide test stream. This error is identified as category (i) in GUM. It can be shown that low level (less than 1,000 ppm) air contaminant causes insignificant error (less than 0.01%) in the compressibility ratio and density ratio used in the derivation of $e_{MUT.}$

Sampling errors

The Triple Point data acquisition system reads temperature and pressure signals at a rate of one sample per second. This relatively high sampling rate ensures that an adequate number of data samples is available for establishing the correct average values for metering error calculation.

Due to the use of an active temperature and pressure control system in Triple Point, the test loop operates in a steady state condition throughout the entire calibration cycle. The category (c) sampling error is considered to be negligible when a steady state operating condition is maintained.

Resolution Errors

Errors caused by inadequate resolution of instruments are identified as category (f) in GUM. All of the Triple Point analog signals are digitized before being processed by the computer system. Assuming s is the minimum registration increment of a digital signal, then the signal can be read to the closest ±½(s). The resolution errors associated with digitized signals with uniform (rectangular) distribution are represented by:

$$u_s = \frac{1}{2} \times \frac{s}{\sqrt{3}} = \frac{s}{2\sqrt{3}} \qquad (19)$$

Instrument	Minimum Digital Increment	Resolution Uncertainty $u_{resolution(C)}$	Calibration Uncertainty $u_{calibration(C)}$	Total Uncertainty $u_{(C)}$
RTD (Temperature)	±0.01ºF	±0.0029ºF	±0.09ºF	±0.09ºF
Diff. Pressure Transducers	±0.1"w.c.	±0.029"w.c.	±0.021"w.c.	±0.036"w.c.
Static Pressure Transducer	±0.1 psia	±0.029 psia	±0.021 psia	±0.036 psia

Table 2 Uncertainties of Triple Point Instruments

Measurement uncertainties caused by inadequate instrument resolution are added to the calibration uncertainty by using the root-sum-square formula:

$$u_c^2(C) = u_{calibration(C)}^2 + u_{resolution(C)}^2 \qquad (20)$$

Long-term Stability and Repeatability

Due to the newness of the Triple Point Facility and process, no long-term stability and repeatability data is available for analysis at the time of writing of this report. In order to maintain a high level of long term system performance, a strict quality control program is planned for the start up and on-going operation of the facility. This control program will be implemented in accordance with the ISO 17025 standard. The key features of this program include:

- Comparison with check meters on a weekly basis. Test records will be kept for each reference meter run in order to build up a statistical database for evaluating the long-term performance of the test loop. The check meters will also be compared against an in-house atmospheric prover in order to confirm their drift characteristics. Control limit will be set at ±0.2% of meter error deviation. Calibration correction will be made if the test results exceed the control limit, and investigation will be conducted to trace the cause of the problem.

- Verification of the calibration of each sensor in the test loop every three months. This data will be used to refine the long-term stability statistic of the sensors.

- Comparison with transfer meters and other reference standards from time to time. This data will provide long-term stability statistics traceable to other recognized facilities.

- Conducting short-term repeatability tests every three months with the check meters. A series of these short-term repeatability test results will form the basis of a long-term repeatability database.

The effectiveness of the start-up control program will be reviewed after twelve months of operation. Adjustments will be made to the program so that it may be adapted to meet the long-term operational requirements of the facility.

Other additional assumptions

A number of practices and assumptions are used to simplify the analysis and to optimize calibration uncertainties in category (i) described in GUM:

1. The minimum volume counts registered by the reference meters are kept above 10,000 so that the ± 1 count error represents less than ±0.01% of the fractional error.

2. The minimum time period for completing a meter calibration cycle is kept above 60 seconds in order to keep the timing error to less than 0.008%.

3. The conductive and radiative heat exchange between the RTD probes and the meter run piping is assumed to be insignificant due to the small temperature gradient from the pipe to the temperature sensing elements.

4. The RDT self-heating effect is considered negligible in comparison to the overall heat balance of the temperature probes.

5. The reference meters and the MUT are assumed to produce smooth and continuous pulse signals.

6. The meter "put-in" and "take-out" effect on measurement uncertainty u_{Rio} is kept small enough to be negligible by using special flange alignment tools.

Sensitivity Coefficient $\left(\frac{\partial f}{\partial x_i}\right)$	Measurement Uncertainty (u_C)	Absolute Value of $\left(\frac{\partial f}{\partial x_i}\right)\times u_C$	$\left(\frac{\partial f}{\partial x_i}\right)^2 \times u_C^2$
$\frac{\partial f}{\partial N_{MUT}} = 0.01\% / pulse$	N_{MUT} ± 1 count	< 0.0100(%)	< 0.0001(%)^2
$\frac{\partial f}{\partial N_{REF}} = -0.01\% / pulse$	N_{REF} ± 1 count	< 0.0100(%)	< 0.0001(%)^2
$\frac{\partial f}{\partial K_{REF}} = 0.15\% / pulse / ft^3$	K_{REF} ± 0.0598 pulse/ft^3	0.1069(%)	0.0114(%)^2
$\frac{\partial f}{\partial T_{MUT}} = -0.22\% /^o F$	T_{MUT} ± 0.09ºF	0.0198(%)	0.0004(%)^2
$\frac{\partial f}{\partial \Delta P_{MUT}} = 0.04\% /'' w.c.$	ΔP_{MUT} ± 0.036" w.c.	0.0014(%)	0.0000(%)^2
$\frac{\partial f}{\partial P_S} << 0.001\% / psia$	P_S ± 0.036 psia	0.0000(%)	0.0000(%)^2
$\frac{\partial f}{\partial T_{REF}} = 0.22\% /^o F$	T_{REF} ± 0.09ºF	0.0198(%)	0.0004(%)^2
$\frac{\partial f}{\partial \Delta P_{REF}} = -0.04\% /'' w.c.$	ΔP_{REF} ± 0.036"w.c.	0.0014(%)	0.0000(%)^2
	$\sum ABS\left(\frac{\partial f}{\partial x_i}\times u_C\right) = 0.1911(\%)$		
$\sum \left(\frac{\partial f}{\partial x_i}\right)^2 \times u_c^2 = 0.0175(\%)^2$		$\sqrt{\sum\left[\left(\frac{\partial f}{\partial x_i}\right)^2 \times u_C^2\right]} = 0.1324(\%)$	

Table 3 Calculation of the Combined Measurement Uncertainty of Triple Point

No uncertainty figure has been assigned for any of the above items.

Uncertainty for a Single Reference Meter

Rewriting (16) and expressing e_{MUT} as a function of the meter parameters and the correction factors:

$$e_{MUT} = f(N_{MUT}, K_{MUT}, N_{REF}, K_{REF}, C_{RE}, C_T, C_P, C_Z) \quad (21)$$

Since the correction factors in (21) can also be expressed as a function of the temperatures and pressures of the gas at the operating condition, equation (21) for a single reference meter test configuration may be expressed as follow:

$$e_{MUT} = f(N_{MUT}, K_{MUT}, N_{REF}, K_{REF}, T_{MUT}, T_{REF}, P_S, \Delta P_{MUT}, \Delta P_{REF}) \quad (22)$$

For a multiple reference meter test configuration, with reference meters 1 to N operating in parallel, we may express e_{MUT} as a function of all of the temperature and pressure variables measured at each reference meter run. Since little is known about the correlation between the different sensors and instruments in these meter runs at this time, we will introduce all of them as independent variables as follow.

$$e_{MUT} = f(N_{MUT}, N_{REF}, K_{REF}, T_{MUT}, \Delta P_{MUT}, P_s, \Delta T_{REF(1)}..\Delta T_{REF(N)}, \Delta P_{REF(1)}..\Delta P_{REF(N)}) \quad (23)$$

Using the general formula for error propagation from GUM (equation 10 on Page 19 of the 1995 GUM document), the combined variance $u_C^2(e_{MUT})$ becomes:

$$\begin{aligned} u_c^2(e_{MUT}) &= \left(\frac{\partial f}{\partial N_{MUT}}\right)^2 u_c^2(N_{MUT}) + \left(\frac{\partial f}{\partial N_{REF}}\right)^2 u_c^2(N_{REF}) + \left(\frac{\partial f}{\partial K_{REF}}\right)^2 u_c^2(K_{REF}) \\ &+ \left(\frac{\partial f}{\partial T_{MUT}}\right)^2 u_c^2(T_{MUT}) + \left(\frac{\partial f}{\partial \Delta P_{MUT}}\right)^2 u_c^2(\Delta P_{MUT}) + \left(\frac{\partial f}{\partial P_S}\right)^2 u_c^2(P_s) \\ &+ \left(\frac{\partial f}{\partial T_{REF(1)}}\right)^2 u_c^2(T_{REF(1)}) .. \left(\frac{\partial f}{\partial T_{REF(N)}}\right)^2 u_c^2(T_{REF(N)}) \\ &+ \left(\frac{\partial f}{\partial \Delta P_{REF(1)}}\right)^2 u_c^2(\Delta P_{REF(1)}) .. \left(\frac{\partial f}{\partial \Delta P_{REF(N)}}\right)^2 u_c^2(\Delta P_{REF(N)}) \end{aligned} \quad (24)$$

where $\partial f / \partial N_{MUT}, \partial f / \partial N_{REF}, \ldots, \partial f / \partial \Delta P_{REF(1)} .. \partial f / \partial \Delta P_{REF(N)}$ etc. of the partial differential equation are the sensitivity coefficients of the independent variables, and the u_c value of each variable is the standard uncertainty associated with that variable. The covariance terms were examined and are ignored at this time in equation (24). The impact of these covariance terms will be examined in a later section.

The sensitivity coefficients $\partial f / \partial x_i$ of the function may be calculated either by solving the differential equation, or by numerically observing the change in e_{MUT} produced by a small change in x_i while holding the remaining quantities constant (GUM 5.1.3). We used the later approach by introducing very small changes in the variables x_i, typically by a magnitude equivalent to the standard uncertainty u of that variable, into the spreadsheet computer model described in a preceding section. The changes in e_{MUT} were noted at different operating flow rates, temperatures, and pressures, with the average values of each of the results recorded in Table 3.

All of the standard uncertainty values $u_{calibration}$ in Table 2 were originally published in manufacturer's specification sheets or calibration certifications with a coverage factor other than 1. These values were converted to standard uncertainties (k = 1, level of confidence = 68.27%) with the appropriate divisors. The mathematical treatment of these variables is explained in a preceding section of this paper.

Applying the standard uncertainties and sensitivity coefficients of each component stated in Table 3 to equation (24), the combined standard uncertainty $u_c(e_{MUT})$ of the Triple Point metering error measurement is derived:

$$u_C(e_{MUT}) = \sqrt{\sum\left[\left(\frac{\partial f}{\partial x_i}\right)^2 \times u_C^2\right]} = 0.1324 \quad \text{(unit in \%)} \quad (25)$$

The standard uncertainty $u_C(e_{MUT})$ is then converted into an expanded uncertainty $U(e_{MUT})$ with a coverage factor k = 2:

$$U(e_{MUT}) = 0.1324(\%) \times 2 = .27 \quad \text{(unit in \%)} \quad (26)$$

Based on the result of (26), we may draw the following conclusion:

The expanded measurement uncertainty of the Triple Point turbine meter calibration facility is ±0.27% of deviation with a confidence level of approximately 95% assuming a normal distribution. This uncertainty figure was developed in accordance with the GUM 1995 document.

Uncertainty of Correlated Variables

According to equation (13), (14) and (15) of the GUM document, the combined variance of a series of coordinated variables is given as:

$$u_c^2(y) = \sum_{i=1}^{N} \left[\frac{\partial f}{\partial x_i}\right]^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j} u(x_i)u(x_j)r(x_i, x_j) \qquad (27)$$

where $r(x_i, x_j)$ is a correlation coefficient which has a value of -1 to 1 depending on the degree of correlation between $u(x_i)$ and $u(x_j)$. The value in the second summation term of equation (27) represents the uncertainty contribution due to the correlation between coordinated variables.

In the Triple Point test loop, we can identify two sets of variables which have a very high probability of being coordinated. These variables are the temperatures T_{REF} and $T_{MUT.}$, and the differential pressures ΔP_{REF} and ΔP_{MUT}. Since the RTDs and pressure sensors were made by the same manufacturers and most likely calibrated using the same reference standards, it is therefore reasonable to assume that each pair of these instruments has a positive correlation coefficient. The exact values of the correlation coefficients are not known, but the maximum value would not exceed +1.

From Table 3, we may also observe that the products of both sets of partial differentials have a negative sign, that is:

$$\left(\frac{\partial f}{\partial T_{REF}}\right)\left(\frac{\partial f}{\partial T_{MUT}}\right) = \text{negative value} \qquad (28)$$

$$\left(\frac{\partial f}{\partial \Delta P_{REF}}\right)\left(\frac{\partial f}{\partial \Delta P_{MUT}}\right) = \text{negative value} \qquad (29)$$

Applying (28) and (29) to equation (27), it can be seen that the summation of the covariance terms in (27) is also negative, and therefore would serve to reduce the combined variance, and hence the uncertainty of the test loop.

In an earlier section, we proposed to ignore the uncertainty contribution of the correlated variables for the present analysis. It is based on the fact that the exact values of the correlation coefficients are not known at this time, and that the inclusion of the correlated variables could only reduce the combined uncertainty of the test loop. Since the focus of this report is to establish a conservative uncertainty estimate for Triple Point, ignoring these covariance terms is consistent with our objective. The degree of correlation between the various sensors can be determined in the future by repeated calibration of these sensors.

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